

NAG Toolbox for MATLAB

f08uf

1 Purpose

f08uf computes a split Cholesky factorization of a real symmetric positive-definite band matrix.

2 Syntax

```
[bb, info] = f08uf(uplo, kb, bb, 'n', n)
```

3 Description

f08uf computes a split Cholesky factorization of a real symmetric positive-definite band matrix B . It is designed to be used in conjunction with f08ue.

The factorization has the form $B = S^T S$, where S is a band matrix of the same bandwidth as B and the following structure: S is upper triangular in the first $(n+k)/2$ rows, and transposed — hence, lower triangular — in the remaining rows. For example, if $n = 9$ and $k = 2$, then

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & & & & & & \\ & s_{22} & s_{23} & s_{24} & & & & & \\ & & s_{33} & s_{34} & s_{35} & & & & \\ & & & s_{44} & s_{45} & & & & \\ & & & & s_{55} & & & & \\ & & & & & s_{66} & & & \\ & & & & s_{64} & s_{65} & s_{66} & & \\ & & & & & s_{75} & s_{76} & s_{77} & \\ & & & & & & s_{86} & s_{87} & s_{88} \\ & & & & & & & s_{97} & s_{98} & s_{99} \end{pmatrix}.$$

4 References

None.

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

Indicates whether the upper or lower triangular part of B is stored.

uplo = 'U'

The upper triangular part of B is stored.

uplo = 'L'

The lower triangular part of B is stored.

Constraint: **uplo** = 'U' or 'L'.

2: **kb** – int32 scalar

If **uplo** = 'U', the number of superdiagonals, k_b , of the matrix B .

If **uplo** = 'L', the number of subdiagonals, k_b , of the matrix B .

Constraint: **kb** ≥ 0 .

3: **bb(ldbb,*)** – double array

The first dimension of the array **bb** must be at least **kb** + 1

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n symmetric positive-definite band matrix B .

The matrix is stored in rows 1 to $k_b + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of B within the band must be stored with element B_{ij} in **bb**($k_b + 1 + i - j, j$) for $\max(1, j - k_b) \leq i \leq j$;
 if **uplo** = 'L', the elements of the lower triangle of B within the band must be stored with element B_{ij} in **bb**($1 + i - j, j$) for $j \leq i \leq \min(n, j + k_b)$.

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The first dimension of the array **bb** and the second dimension of the array **bb**. (An error is raised if these dimensions are not equal.)

n , the order of the matrix B .

Constraint: $n \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldbb

5.4 Output Parameters

1: **bb(ldbb,*)** – double array

The first dimension of the array **bb** must be at least **kb** + 1

The second dimension of the array must be at least $\max(1, \mathbf{n})$

B contains the elements of its split Cholesky factor S .

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **kb**, 4: **bb**, 5: **ldbb**, 6: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** = i , the factorization could not be completed, because the updated element $b(i, i)$ would be the square root of a negative number. Hence B is not positive-definite. This may indicate an error in forming the matrix B .

7 Accuracy

The computed factor S is the exact factor of a perturbed matrix $(B + E)$, where

$$|E| \leq c(k+1)\epsilon |S^T| |S|,$$

$c(k+1)$ is a modest linear function of $k+1$, and ϵ is the *machine precision*. It follows that $|e_{ij}| \leq c(k+1)\epsilon \sqrt{(b_{ii}b_{jj})}$.

8 Further Comments

The total number of floating-point operations is approximately $n(k+1)^2$, assuming $n \gg k$.

A call to f08uf may be followed by a call to f08ue to solve the generalized eigenproblem $Az = \lambda Bz$, where A and B are banded and B is positive-definite.

The complex analogue of this function is f08ut.

9 Example

```
uplo = 'L';
kb = int32(1);
bb = [2.07, 1.69, 0.65, 1.17;
      0.95, -0.29, -0.33, 0];
[bbOut, info] = f08uf(uplo, kb, bb)

bbOut =
    1.4387    1.0502    0.7463    1.0817
    0.6603   -0.3886   -0.3051         0
info =
         0
```